The Schwinger Model with Perfect Staggered Fermions

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We construct a new perfect action for free staggered fermions, which is more local than the one obtained from the standard block average scheme. This pays off in superior properties after a short ranged truncation. This action is "gauged by hand" and tested in Schwinger model simulations by means of a new variant of hybrid MC. Using "fat links" for the gauge field, we obtain a tiny "pion" mass down to $\beta \lesssim 1.5$, and the "eta" mass follows very closely the prediction of asymptotic scaling.

It has been observed that a good implementation of classically perfect actions provides an excellent level of improvement in the sense that lattice artifacts are drastically suppressed. However, the most successful applications have been limited to two dimensions so far, and they involve a huge number of couplings [1,2]. This implies that corresponding 4d actions are hardly applicable. Therefore it is crucial to achieve good improvement with *rather few extra terms*, and also this can be studied first in $d = 2$. However, the known classically perfect actions of Refs. [1,2] are not local enough for a very short ranged truncation to be sensible, hence – in view of $d = 4$ – different construction methods should be considered. Here we present a successful application of "gauging by hand", i.e. we use the truncated perfect couplings of the free fermion and insert gauge couplings by hand. This leads to a relatively modest overhead in the number of couplings compared to the standard action, but to an improvement on the same level as classical perfection. We conclude that this procedure is promising for $d = 4$.

Perfect actions are constructed by iterating block variable renormalization group transformations (RGTs). For massless staggered fermions, the RGT should not mix the (pseudo-)flavors in order to preserve the remnant chiral symmetry $U(1) \otimes U(1)$. This can be achieved by blocking each flavor separately [3]. As an improvement over the standard block average (BA) scheme [4], we propose "**partial decimation**" (PD) [5]. The free propagator Δ_{BA} is built by averaging source and sink in each block, whereas in the PD scheme one only averages the source and treats the sink by decimation, or vice versa. Both lead to local perfect actions for free staggered fermions, and in both cases we optimize the locality by tuning the RGT parameters. The optimization criterion is that the mapping down to $d = 1$, $\Delta^{-1}(p_1, 0, \dots, 0)$, only couples nearest neighbors. For the PD scheme this is the case for a simple δ function blocking at $m = 0$ [5]. However, that criterion can be fulfilled at arbitrary mass m. In $d > 1$ the PD scheme leads to a superior degree of locality, i.e. to a faster exponential decay of the couplings. Hence the truncation can be expected to be less harmful.

The truncation itself should also be done with some care. We found **mixed periodic boundary conditions** to be optimal: we impose *anti-periodicity* over 6 lattice spacings in the direction of odd coupling distance, and *periodicity* over 6 lattice spacings in the other direction(s). The resulting couplings in $d = 2$ are given in the following Table; for $d = 4$ we refer again to Ref. [5]. There we also discuss massive fermions, including the case of non-degenerated flavors.

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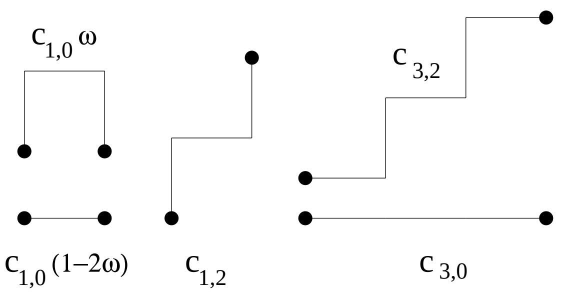
$m=0$	BA	PD
$c_{1,0}$	0.866299	**0.878234**
$c_{1,2}$	0.066850	**0.060883**
$c_{3,0}$	0.016043	**0.010425**
$c_{3,2}$	-0.008022	**-0.005212**

The better quality of the truncated perfect action arising from the PD scheme is confirmed by the dispersion relation and by the thermodynamic scaling behavior [5,6]. In the latter case, the scaling region is extended by one order of magnitude compared to the standard action.

In our **application to the Schwinger model** we obtain a "π" and a "η" particle, since we deal with two massless flavors. We use the couplings of the PD scheme in the above Table, and we include the following lattice paths:



The staple weight ω is tuned so that the smallest absolute values of fermion matrix eigenvalues are minimized. We find $\omega = \mathbf{0.238}$ to be optimal, and we use the resulting fat link.

For the pure gauge part, we use an ultralocal plaquette action, which is perfect in $d=2$ [7,8].

In our action, a fixed site couples to 24 other sites, which is an overhead of a factor of 6 compared to the standard action. In view of QCD, we design an algorithm which avoids a proportional increase in compute time:

Variant of Hybrid Monte Carlo
(1) Possible Molecular Dynamics steps are identified using the standard action.
(2) The acceptance decision is based on the quasi-perfect action.

The crucial question is the acceptance rate; it is good down to a minimal value of β. If we include the fat link in (1), then it works well down to very small β ($\beta \lesssim 1.5$). Still the simplification in (1) accelerates the algorithm by a factor of ≈ 4, while the acceptance rate is divided by ≈ 2.

We generate $3 \cdot 10^5$ configurations on a $\mathbf{16 \times 16}$ **lattice**, and we measure after multi-steps of 100 configurations [3]. The simulation results [8] are unquenched.

The **spectrum** is determined from local bilinears [9]:

$$M^{1-}_{--}(x) = (-1)^{x_1+x_2}[\bar{\psi}_x \psi_{x+(1,0)} - \bar{\psi}_{x+(1,0)}\psi_x]$$
$$M^{1+}_{++}(x) = \bar{\psi}_x \psi_{x+(1,0)} + \bar{\psi}_{x+(1,0)}\psi_x \ .$$

We measure the correlators $C^{\alpha}_{\sigma_1\sigma_2}(p_1, x_2) =$

$$\sum_{x_1} \langle M^{\alpha}_{\sigma_1\sigma_2}(x_1,x_2) M^{\alpha}_{\sigma_1\sigma_2}(0,0)\rangle e^{ix_1p_1} \ .$$

- The isovector current correlation is obtained from C^{1-}_{--}. It has only connected contributions and yields the "pion".
- The isoscalar current correlation is obtained from C^{1+}_{++}. It has disconnected contributions too and yields the "eta particle".

At $\beta = 3$ we evaluate the dispersion relation $E^2(p_1) = m^2 + p_1^2$ from cosh fits for m_π and m_η. The result is shown in Fig. 1. Then we measure these masses with the standard action (from $p_1 = 0$) and with the quasi-perfect action (from $p_1 = 0$ and $p_1 = \pi/8$), see Fig. 2. The continuum predictions are $m_\pi = 0$ (**scaling**), $m_\eta^2 = 2/(\pi\beta)$ (**asymptotic scaling**).

Using the quasi-perfect action described above, we find a very small m_π, i.e. very good scaling, down to $\beta \lesssim 1.5$. The η dispersion follows very closely the prediction of asymptotic scaling down to $\beta \approx 1$. These results — in particular m_π, which tests the level of improvement — are even better than those of Ref. [2], which used truncated perfect Wilson fermions together with a

[3]The number 100 corresponds to the maximal autocorrelation time for topological observables (for other quantities it is ≈ 25).

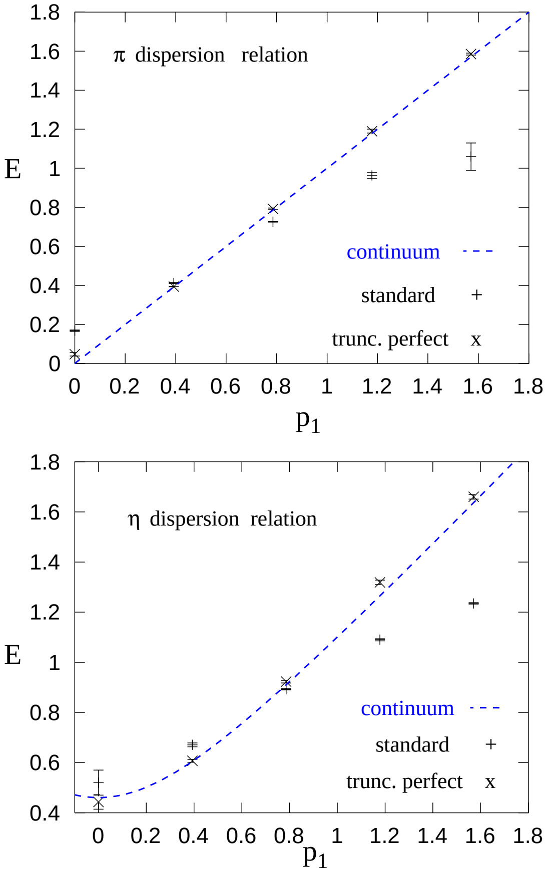


Figure 1. “Meson” dispersion relations at $\beta = 3$.

classically perfect vertex function (parameterized by 123 independent couplings).

This shows that a truncated perfect free fermion, which is suitably “gauged by hand”, *can* represent a highly improved, short-ranged action. This program is applicable and promising for QCD [10].

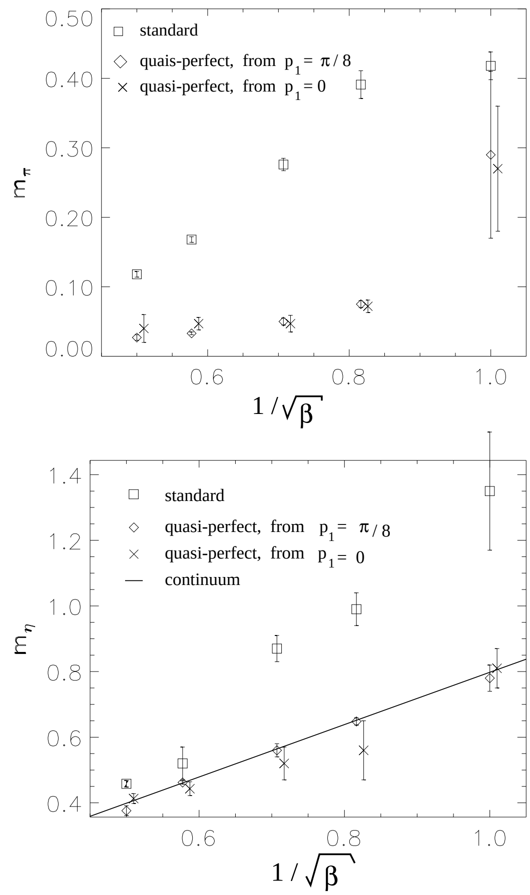


Figure 2. The masses m_π and m_η measured at various values of β.

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